# Orientifold and stringy frame of the Kerr Spinning Particle

Alexander Burinskii

NSI Russian Academy of Sciences

B. Tulskaya 52, 115191 Moscow, Russia

February 1, 2008

### Abstract

We show that the frame of the Kerr spinning particle consists of two topologically coupled strings. One of them is the Kerr singular ring representing a string with an orientifold world-sheet. It has electromagnetic excitations (traveling waves), which corresponds to the old model of the Kerr's microgeon. The excitations induce the appearance of the extra axial string which is topologically coupled to the Kerr circular string and is a carrier of pp-waves and fermionic zero modes. This string can be described by the Witten field model for superconducting strings.

## 1  Introduction

The Kerr rotating black hole solution displays some remarkable relations to spinning particles [1, 2, 3, 4, 5, 6, 7, 8]. For the parameters of elementary particles, $|a| >> m$, the black-hole horizons disappear, obtaining the source in the form of a closed singular ring of the Compton radius $a = J/m$. In the old model of the Kerr spinning particle [2] this ring was considered as a gravitational waveguide containing traveling electromagnetic (and fermionic) wave excitations. The assumption that the Kerr singular ring represents a closed relativistic string was advanced about thirty years ago [3], which has got confirmation on the level of the evidences [4, 3]. However, the attempts to show it explicitly ran into obstacles. The motion of the Kerr ring is the lightlike sliding along itself, which could be described as a string containing lightlike modes of only one direction. However, the string equations do not admit such solutions. This problem can be resolved by introducing an orientifold world-sheet [8].

Another type of excitations is a vibration of the Kerr circular string. It is connected with a complex structure of the Kerr geometry [9]. Evolution of the complex Kerr,s retarded-time parameter $\tau = t + \sigma$ forms a world-sheet. It corresponds to a hyperbolic string which is very similar to the $N = 2$ string. It was shown that consistence of this string demands also the orientifold world-sheet structure [5].

The novel feature appears when we consider the exact solutions for electromagnetic excitations of the Kerr circular string. One obtains the appearance of the extra axial string which is connected with the NUT parameter and is infinite (for a free particle). This string is related to the Dirac monopole string and is topologically coupled to the Kerr circular string. It acquires the traveling pp-waves which are induced by excitations of the Kerr string and can be described by the Witten field model for superconducting strings. We conjecture that this string may play the role of a carrier of the wave function.

Our treatment is based on the Kerr-Schild formalism [10] and the results of previous paper [9] where the real and complex structures of the Kerr geometry were considered.

## 2  Orientifold structure of the Kerr string

The Kerr-Schild approach [10] is based on the Kerr-Schild form of the metric: $g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu$ where $\eta_{\mu\nu}$ is the metric of auxiliary Minkowski space-time, $h = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}$, and $k_\mu$ is a twisting null

field, which is tangent to the Kerr principal null congruence (PNC) which is determined by a twisted 1-form [1] The field $k^\mu$ is null with respect to $\eta_{\mu\nu}$, as well as with respect to the full metric $g_{\mu\nu}$.

The Kerr singular ring is the branch line of the Kerr space on two folds: positive sheet ($r > 0$) and 'negative' one ($r < 0$). Since for $|a| >> m$ the horizons disappear, there appears the problem of the source of the Kerr solution with the alternative: either to remove this twofoldedness or to give it a physical interpretation.

The negative sheet can be retained if we shall treat it as the sheet of advanced fields. In this case the source of the spinning particle turns out to be the Kerr singular ring with the electromagnetic excitations in the form of traveling waves, which generate spin and mass of the particle. A model of this sort was suggested long ago as a "microgeon with spin" [2], where the Kerr singular ring was considered as a waveguide providing a circular propagation of the electromagnetic or fermionic wave excitations. The lightlike structure of the Kerr ring is seen from the analysis of the PNC which contains tangent to the ring lightlike rays. It is seen from the form of $k^\mu$. Approaching the ring ($r \to 0$) PNC takes the form $k \simeq dt - (xdy - ydx)/a = dt - ad\phi$, and the light-like vector field $k_\mu$ turns out to be tangent to the world sheet of the Kerr ring. It shows that the Kerr ring is sliding along itself with the speed of the light. It was recognized long ago [3] that the Kerr singular ring can be considered as the string having the traveling wave excitations in the model of microgeon. The Kerr twofoldedness shows that it is the "Alice" type of string. One of the evidences of its stringy structure was obtained by the analysis of the axidilatonic generalization of the Kerr solution (given by [11]). It was shown [4] that the fields near the Kerr ring are similar to the field around a heterotic string.

The world-sheet of the Kerr ring satisfies the string wave equation and constraints; however, there appear the problem with boundary conditions. Representing solution as the sum of the 'left' and 'right' modes

$$X_R^\mu(\tau - \sigma) = \frac{1}{2}[x^\mu + l^2 p^\mu(\tau - \sigma) + il\sum_{n\neq 0}\frac{1}{n}\alpha_n^\mu e^{-2in(\tau-\sigma)}], \tag{1}$$

$$X_L^\mu(\tau + \sigma) = \frac{1}{2}[x^\mu + l^2 p^\mu(\tau + \sigma) + il\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}_n^\mu e^{-2in(\tau+\sigma)}], \tag{2}$$

one sees that the string constraints $\dot{X}_\mu \dot{X}^\mu + X'_\mu X'^\mu = 0$, $\dot{X}_\mu X'^\mu = 0$ [$()' \equiv \partial_\sigma()$], are satisfied if the modes are lightlike $(\partial_\sigma X_{L(R)\mu})(\partial_\sigma X_{L(R)}^\mu) = 0$.

Indeed, setting $2\sigma = a\phi$ one can describe the lightlike worldsheet of the Kerr ring (in the rest frame of the Kerr particle) by the lightlike surface

$$X_L^\mu(t, \sigma) = x^\mu + \frac{1}{\pi T}\delta_0^\mu p^0(t + \sigma) + \frac{a}{2}[(m^\mu + in^\mu)e^{-i2(\tau+\sigma)} + (m^\mu - in^\mu)e^{i2(\tau+\sigma)}], \tag{3}$$

where $m^\mu$ and $n^\mu$ are two spacelike basis vectors lying in the plane of the Kerr ring. One can see that $\partial_\sigma X_L^\mu$ will be a light-like vector if one sets $p^0 = 2\pi aT$.

Therefore, the Kerr world-sheet could be described by modes of one (say "left") null direction. However, one sees that the closed string boundary condition $X^\mu(\tau, \sigma) = X^\mu(\tau, \sigma + \pi)$ will not be satisfied since the time component $X_L^0(t, \sigma + \pi)$ acquires contribution from the second term in (3), which is usually compensated by this term from the 'right' mode. Therefore, the strings having only the "left" modes cannot be closed.

This obstacle can be removed by the formation of the world-sheet orientifold. The interval of an open string $\sigma \in [0, \pi]$ is formally extended to $[0, 2\pi]$, setting

$$X_R(\sigma + \pi) = X_L(\sigma), \qquad X_L(\sigma + \pi) = X_R(\sigma). \tag{4}$$

By such an extension, the both types of modes, "right" and "left", appear since the "left" modes play the role of "right" ones on the extended piece of interval. If the extension is completed by the

---

[1] The rays of the Kerr PNC are twistors and the Kerr PNC is determined by the Kerr theorem, see detailed description in [9].
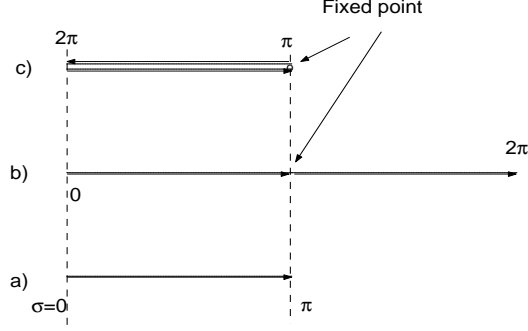
Figure 1: Formation of orientifold: a) the initial string interval; b) extension of the interval and formation of the both side movers; c) formation of the orientifold.

changing of orientation on the extended piece, $\sigma' = \pi - \sigma$, with a subsequent identification of $\sigma$ and $\sigma'$, then one obtains the closed string on the interval $[0, 2\pi]$ which is folded and takes the form of the initial open string.

Formally, the worldsheet orientifold represents a doubling of the worldsheet with the orientation reversal on the second sheet. The fundamental domain $[0, \pi]$ is extended to $\Sigma = [0, 2\pi]$ with formation of folds at the ends of the interval $[0, \pi]$.

# 3  Solutions of the e.m. field equations

Field equations for Einstein-Maxwell system in the Kerr-Schild class were obtained in [10]. We will concentrate here on the electromagnetic excitations of the Kerr ring. These e.m. fields are aligned to the Kerr PNC and described by the tetrad components of the self-dual tensor $\mathcal{F}_{12} = AZ^2$, $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$. The field equations are

$$A_{,2} - 2Z^{-1}\bar{Z}Y_{,3}\, A = 0, \tag{5}$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma_{,2} - Z^{-1}Y_{,3}\,\gamma = 0. \tag{6}$$

For the sake of simplicity we consider the gravitational Kerr-Schild field as stationary. The corresponding oscillating solutions can be obtained by introduction of a complex retarded time parameter $\tau = t_0 + i\sigma = \tau|_L$ which is determined as a result of the intersection of the left (L) null plane and the complex world line (for details see [9]). The left null planes are the left generators of the complex null cones and play the role of null cones in the complex retarded-time construction. The $\tau$ parameter satisfies to relations $(\tau)_{,2} = (\tau)_{,4} = 0$. It allows one to represent the equation (5) in the form $(AP^2)_{,2} = 0$, and to get the following general solution

$$A = \psi(Y, \tau)/P^2. \tag{7}$$

Action of the operator $\mathcal{D}$ on the variables $Y, \bar{Y}$ and $t_0$ is following $\mathcal{D}Y = \mathcal{D}\bar{Y} = 0$, $\mathcal{D}t_0 = P^{-1}$. The equation (6) takes the form $\partial_{t_0} A = -(\gamma P)_{,\bar{Y}}$. For stationary background $P = 2^{-1/2}(1 + Y\bar{Y})$, and $\dot{P} = 0$. The coordinates $Y$, and $\tau$ are independent from $\bar{Y}$, which allows us to integrate this equation and we obtain the following general solution

$\gamma = -P^{-1}\int \dot{A}d\bar{Y} = -P^{-1}\dot{\psi}(Y,\tau)\int P^{-2}d\bar{Y} = \frac{2^{1/2}\dot{\psi}}{P^2 Y} + \phi(Y,\tau)/P$, where $\phi$ is an arbitrary analytic function of $Y$ and $\tau$.

The term $\gamma$ in $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$ describes a part of the null electromagnetic radiation which falls of asymptotically as $1/r$ and propagates along the Kerr principal null congruence $e^3$. As it was discussed in [9, 8], this field acquires interpretation of the vacuum zero point field which has to be regularized similar to the zero point field in the Casimir effect.

3

# 4  Axial strings

Considering the second term in $\mathcal{F}_{31}$, we obtain the terms containing the factors which depend on coordinate $Y = e^{i\phi}\tan\frac{\theta}{2}$ and turn out to be singular at the axis of symmetry (z-axis).

These factors result the appearance of two half-infinite lines of singularity, $z^+$ and $z^-$, which correspond to $\theta = 0, \quad Y \to 0$ and $\theta = \pi, \quad Y \to \infty$ and coincide with corresponding axial lightlike rays of the Kerr PNC. On the "positive" sheet of the Kerr background these two half-rays are directed outward. They are going from the "negative" sheet and appear on the "positive" sheet passing through the Kerr ring (see Fig. 2).
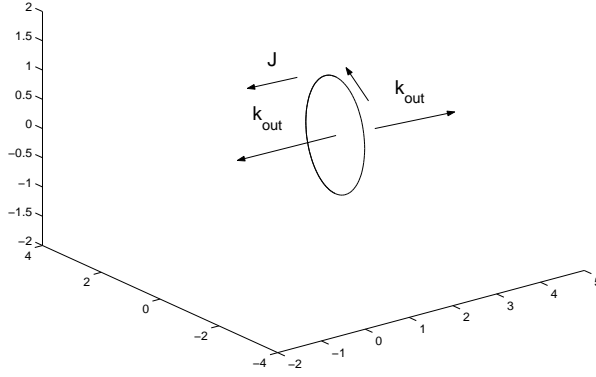


Figure 2: Schematic description of the Kerr spinning particle. Circular string and two axial half-infinite strings directed outwards.

We will be interested in the wave terms and omit the terms describing the longitudinal components and the field $\gamma$. By using the expressions for the Kerr tetrad we obtain that the wave terms of the e.m. field take the form[2]

$$\mathcal{F}|_{wave} = f_R \; d\zeta \wedge du + f_L \; d\bar{\zeta} \wedge dv, \tag{8}$$

where the factor $f_R = (AZ)_{,1}$ describes the "right" waves propagating along the $z^+$ half-axis, and the factor $f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1}$ describes the "left" waves propagating along the $z^-$ half-axis.

Since $Z/P = (r + ia\cos\theta)^{-1}$, all the terms are also singular at the Kerr ring $r = \cos\theta = 0$. Therefore, the singular excitations of the Kerr ring turn out to be connected with the axial singular waves.

Traveling waves along the Kerr ring are generated by the function $\psi_n(Y,\tau) = qY^n \exp i\omega_n\tau \equiv q(\tan\frac{\theta}{2})^n \exp i(n\phi + \omega_n\tau)$ which has near the Kerr ring the form $\psi = \exp i(n\phi + \omega t)$, and index $|n|$ corresponds to the number of the wave lengths along the Kerr ring. The factor $Y^n$ leads to the appearance of the $z^\pm$ axial singularities. When considering asymptotical properties of these singularities by $r \to \infty$, we have $z = r\cos\theta$, and for the distance $\rho$ from the $z^+$ axis we have the expression $\rho = z\tan\theta \simeq 2r|Y|$ by $Y \to 0$. Therefore, for the asymptotical region near the $z^+$ axis we have to put $Y = e^{i\phi}\tan\frac{\theta}{2} \simeq e^{i\phi}\frac{\rho}{2r}$, and $|Y| \to 0$, while for the asymptotical region near the $z^-$ axis $Y = e^{i\phi}\tan\frac{\theta}{2} \simeq e^{i\phi}\frac{2r}{\rho}$, and $|Y| \to \infty$. The parameter $\tau = t - r - ia\cos\theta$ takes near the z-axis the values $\tau_+ = \tau|_{z^+} = t - z - ia$, and $\tau_- = \tau|_{z^-} = t + z + ia$.

For $|n| > 1$ the solutions contain the axial singularities which do not fall of asymptotically and are increasing. Therefore, the treatment has to be restricted by the cases $|n| \le 1$. The leading singular wave for $n = 1$, $\mathcal{F}_1^-$, propagates to $z = -\infty$ along the $z^-$ half-axis, and the leading singular wave for

---

[2]Here the null coordinates are used $2^{\frac{1}{2}}\zeta = x + iy, \quad 2^{\frac{1}{2}}\bar{\zeta} = x - iy, \quad 2^{\frac{1}{2}}u = z - t, \quad 2^{\frac{1}{2}}v = z + t.$

4

$n = -1$, $\mathcal{F}_-^+1$ propagates to $z = +\infty$ along the $z^+$ half-axis. They have the form

$$\mathcal{F}_1^- = \frac{4qe^{i2\phi+i\omega_1\tau_-}}{\rho^2}\, d\bar{\zeta} \wedge dv, \qquad \mathcal{F}_{-1}^+ = -\frac{4qe^{-i2\phi+i\omega_{-1}\tau_+}}{\rho^2}\, d\zeta \wedge du, \qquad (9)$$

The waves with $n = 0$, $\mathcal{F}_0^{\pm}$, are regular.

One sees that the partial solutions turns out to be asymmetric with respect to the $z^{\pm}$ half-axis, which can lead to a nonstationarity via a recoil. This e.m. field can also be obtained from the potential $\mathcal{A} = -AZe^3 - \chi d\bar{Y}$, where $A = \psi/P^2$ and $\chi = \int P^{-2}\psi dY$, and $\bar{Y}$ being kept constant in this integration.

Each of the partial solutions represents the singular plane-fronted e.m. wave propagating along $z^+$ or $z^-$ half-axis without damping. It is easy to point out the corresponding self-consistent solution of the Einstein-Maxwell field equations which belongs to the well known class of pp-waves [12]. These singular pp-waves propagate outward along the $z^+$ and/or $z^-$ half-axis and can be regularized by a Higgs field leading to the axial stringy currents.

By the analysis of the corresponding gravitational field equations, one can see that the resulting metric acquires an imaginary contribution to the mass term, which is the evidence that the NUT parameter has to be involved, and also that the metric is going out of the Kerr-Schild class. Therefore, the obtained singular strings are related to the Dirac monopole string.

To exclude the monopole charge and to get a symmetric stringy solution, one has to use a combination of the $n = \pm 1$ excitations. There is one solution containing combination of three terms with $n = -1, 0, 1$ with $\omega_1 = -\omega_{-1} = \omega$ and $\omega_0 = 0$. It represents especial interest since it has an electric charge and a smooth e.m. field having one half of the wavelength packed along the Kerr string.

Note, that orientifold structure of the Kerr circular string admits apparently the excitations with $n = \pm 1/2$ too, so far as the negative half-wave can be packed on the covering space turning into positive one on the second sheet of the orientifold. However, this question demands an additional consideration. The above axial half-infinite strings are carriers of pp-waves and for the moving particles these pp-waves are modulated by de Broglie periodicity. It allows one to conjecture that this string could be a carrier of the wave function.

At first sight these half-infinite strings looks very strange. However, they are half-infinite (like the Dirac monopole string) only for the free particles. By interaction they can form the closed quantized loops. Such a sort of the linked strings was considered by Garriga and Vachaspati [14]. For the field description of coupled strings the Witten field model of superconducting strings [13] has to be used. The stringy Kerr source model opens an interesting way for topological realizations of the standard model of particle physics and gives a new view of some quantum phenomena. Application of the supersymmetric version of the Witten field model to the Kerr baglike source was discussed in [7].

## Acknowledgments

## References

[1] B. Carter, Phys. Rev. **174** (1968) 1559. W. Israel, Phys. Rev. **D2** (1970) 641. C.A. López, Phys. Rev. **D30** (1984) 313.

[2] A.Ya. Burinskii, Sov. Phys. JETP, **39**(1974)193.

[3] D. Ivanenko and A.Ya. Burinskii, Izvestiya Vuzov Fiz. n.5 (1975) 135 (in russian).

[4] A. Burinskii, Phys.Rev.**D 52** (1995)5826, hep-th/9504139.

[5] A.Ya. Burinskii, Phys.Lett. **A 185** (1994) 441; *String-like Structures in Complex Kerr Geometry.* In: "Relativity Today", Edited by R.P.Kerr and Z.Perjés, Akadémiai Kiadó, Budapest, 1994, p.149, hep-th/9303003.

[6] A. Burinskii, Phys.Rev.**D 57** (1998)2392, hep-th/9704102. Class. Quant. Grav. **16**(1999)3497, hep-th/9903032.

[7] A. Burinskii, Grav.& Cosmology. **8** (2002) 261, hep-th/9910045.

[8] A. Burinskii, Phys.Rev.**D68**(2003)105004, hep-th/0308096.

[9] A. Burinskii, Clas. Quant. Grav. **20** (2003)905; Phys.Rev. **D 67** (2003) 124024, gr-qc/0212048.

[10] G.C. Debney, R.P. Kerr, A.Schild, J. Math. Phys. **10**(1969) 1842.

[11] A. Sen, Phys. Rev. Lett. **69** (1992) 1006, Nucl.Phys.,**B 388** (1992) 457.

[12] D.Kramer, H.Stephani, E. Herlt, M.MacCallum, "Exact Solutions of Einstein's Field Equations", Cambridge Univ. Press, 1980.

[13] E. Witten, Nucl.Phys.,**B249**(1985)557.

[14] J. Garriga and T. Vachaspati, Nucl.Phys.,**B438**(1995)161, hep-ph/9411375.